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ACCURACY COMPARISON OF MULTIVARIATE NEWTON-RAPHSON AND NEWTON-KANTOROVICH METHODS THROUGH NUMERICAL SIMULATION IN NONLINEAR SYSTEMS

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ABSTRACT

Nonlinear systems of equations often appear in various fields of science and generally cannot be solved analytically, so numerical methods are required. However, previous studies have not provided a direct comparison of the accuracy and efficiency of the Multivariate Newton-Raphson method and the Newton-Kantorovich method when applied to the same nonlinear system, creating a gap in understanding their relative performance. This study aims to analyze and compare the performance of two numerical methods, namely the Newton-Raphson method and the Newton-Kantorovich method, in solving nonlinear systems of equations numerically. The evaluation is based on the convergence rate, result accuracy, and iteration efficiency of each method. The nonlinear system used involves trigonometric, exponential, and polynomial functions. Simulations were conducted twice using three equations directly for each method. The error tolerance was set at 0.001, with a maximum of 100 iterations. The simulation results showed that the Multivariate Newton-Raphson method had the best performance, requiring only 7 iterations to achieve convergence with a very small error of 2.711×10^{-7}. *In contrast, the Newton-Kantorovich method required 21 iterations and produced an error of* 6.770×10^{-5}, *indicating slower convergence and lower efficiency. Based on these results, it can be concluded that the Multivariate Newton-Raphson method is the more accurate and efficient method for solving nonlinear systems of equations through numerical simulation. This finding contributes to the selection of an appropriate numerical method and opens opportunities for further exploration in higher-dimensional systems.*

Keywords: *multivariate newton-raphson, newton-kantorovich, nonlinear systems of equations, numerical methods.*

I. Introduction

A nonlinear system of equations is a set of equations containing variables with nonlinear relationships, such as powers greater than one, exponential functions, logarithmic functions, or trigonometric functions [1]. Unlike linear systems of equations, which have exact solutions and can be solved using conventional algebraic methods, nonlinear systems generally do not have closed-form solutions and require numerical approaches to obtain their solutions [2]. In various fields such as applied mathematics, engineering, physics, and economics, nonlinear systems of equations often emerge naturally as models of complex phenomena [3]. The high level of complexity in their form and solution behavior makes these systems difficult to solve without computational technology. Therefore, numerical approaches have become the primary method used to solve nonlinear systems of equations in various modern scientific and engineering practices [4].

The multivariate Newton-Raphson method is an extension of the single-variable Newton-Raphson method applied to nonlinear systems of equations with more than one variable [5]. This method uses a linearization approach for nonlinear functions through a first-order Taylor series, resulting in a system of linear equations solved iteratively [6]. At each iteration, the partial derivatives of each function with respect to each variable must be calculated and arranged in the form of a Jacobian matrix [7]. The accuracy and efficiency of this method depend heavily on the ability to calculate the Jacobi-an and the selection of appropriate initial values [8]. The main advantage of this method lies in its convergence speed when the initial value is sufficiently close to the actual solution. However, it also has the drawback of being sensitive to initial conditions and the potential for convergence failure in highly complex systems [9].

The Newton-Kantorovich method is a generalization of the Newton method developed on the basis of operator theory and functional analysis [10]. Unlike the Newton-Raphson method, which is typically applied in finite-dimensional spaces, the Newton-Kantorovich method utilizes a Banach space-based approach and Fréchet operators, enabling its application to a broader range of systems, including those defined in infinite-dimensional function spaces [11]. This approach provides a guarantee of local convergence under certain conditions and is suitable for handling nonlinear systems with more complex structures [12]. This method tends to be more complex as it requires explicit operator formulations and often necessitates more iterations [13]. Although theoretically stronger, computational implementation challenges remain a key aspect in the application of this method [14].

Previous studies have demonstrated the significant contribution of the multivariate Newton-Raphson method and Newton-Kantorovich method in solving nonlinear systems of equations through numerical simulation [15], [16], [17]. Both methods have been examined in terms of iteration efficiency, convergence stability, and application flexibility, particularly in handling large-scale and high-dimensional nonlinear systems. [15] found that both methods show strong potential for solving complex numerical problems, particularly in dynamic systems and mathematical analysis, where convergence speed and sensitivity to initial values significantly affect method performance. The multivariate Newton-Raphson method has proven more effective in break-even point analysis because it produces smaller errors and requires fewer iterations than other numerical methods [16]. Its effective-ness in approximating the roots of various types of nonlinear equations, such as polynomial, trigonometric, and exponential equations, is further supported by the findings of [17].

The development of the multivariate Newton-Raphson method continues to demonstrate its relevance in solving more complex nonlinear systems of equations, including through the application of multiple iterative strategies with higher convergence orders [18]. This approach is designed to accelerate the root-finding process without sacrificing accuracy. A study by [19] shows that the multivariate Newton-Raphson method remains stable during the convergence process, even in high-dimensional systems, provided that the initial values are sufficiently close to the actual roots. [20] reports that this method can achieve convergence in just 5 iterations with a final error of 1.72×10^{-8}, demonstrating high efficiency and accuracy compared to other numerical methods in similar scenarios. These findings show that the multivariate Newton-Raphson method is not only effective in simple systems but also reliable in solving large-scale and complex numerical problems.

On the other hand, the Newton-Kantorovich method demonstrates good performance in solving nonlinear systems of equations that require high stability and flexibility in analysis. [21] show that this method remains stable and computationally efficient, even in forms of approximation that are not entirely accurate, particularly in the application of nonlinear predictive control. [22] developed a semi-local convergence analysis to enhance the method's ability to handle weak initial guesses, while [23] introduced the Center-Lipschitz condition to relax convergence requirements in Banach spaces. [24] demonstrated that this method remains reliable in complex control systems despite requiring significant computation. [25] used the Fréchet operator to avoid explicit Jacobian computation, although this resulted in an increase in the number of iterations. [26] reported that the Newton-Kantorovich method can solve parameterized nonlinear systems in just seven iterations with a final error rate of 2.31×10^{-7}, reinforcing the effectiveness of this method in various complex numerical problems.

Although various studies have proven the effectiveness of the Multivariate Newton-Raphson and Newton-Kantorovich methods in solving nonlinear systems of equations, most previous research has focused on algorithm development or on applying these methods separately in specific contexts. The main problem addressed in this study is the lack of a direct and systematic comparison between these two methods when applied to the same nonlinear system under identical conditions, which makes it difficult to determine which method is more accurate and efficient in practice. Few studies have di-rectly compared the accuracy levels of these two methods in solving the same nonlinear system through a controlled numerical simulation approach. Therefore, this study aims to compare the accuracy of the Multivariate Newton-Raphson and Newton-Kantorovich methods in solving nonlinear systems of equations by considering the number of iterations, convergence stability, and final error values through numerical simulation. The contribution of this research lies in providing an empirical benchmark that highlights the relative strengths and weaknesses of both methods. The novelty of this work is the direct head-to-head evaluation of Newton-Raphson and Newton-Kantorovich under the same problem setting, which has rarely been explored in previous studies. While earlier works have mainly emphasized algorithmic development or isolated applications, this study is offers a more explicit comparative perspective by systematically evaluating both methods under identical problem settings through controlled numerical simulation. The results of this study are expected to contribute to the selection of more appropriate and efficient methods based on the characteristics of the nonlinear systems encountered.

II. Research Method

This study uses a quantitative approach through numerical simulation to compare the accuracy of the multivariate Newton-Raphson method and the Newton-Kantorovich method in solving nonlinear systems of equations [27]. The research is designed as a comparative study with a numerical experimental approach, in which both methods are systematically implemented on a predetermined nonlinear system of equations [28]. The equations used consist of three nonlinear functions with three variables, involving trigonometric, exponential, and polynomial elements, to represent the complexity of high-dimensional nonlinear systems.

The algorithm was implemented using the latest version of MATLAB software, with evaluation criteria including (1) the absolute error value relative to the exact solution, (2) the number of iterations until convergence, and (3) computation time. The initial guess was set identically for both methods to ensure the validity of the comparison. The convergence criteria were based on an error tolerance of 0.001 and a maximum of 100 iterations. The results of each method were analyzed descriptively and quantitatively to assess the level of accuracy and numerical efficiency produced. All simulation and data processing procedures were carried out in a controlled manner so that the comparison results could be presented objectively and measurably. The details of the two methods used are explained below.

A. Multivariate Newton-Rapshon Method

The Multivariate Newton-Raphson Method is an iterative technique widely used to solve multivariable nonlinear systems of equations. This method uses a linear approximation based on Taylor series expansion and involves the Jacobian matrix, which contains the partial derivatives of the function with respect to the relevant variables. Updating the Jacobian matrix at each iteration allows the method to adjust the search direction dynamically, thereby accelerating convergence, especially when the initial guess is sufficiently close to the desired solution. However, this advantage is offset by the relatively high computational load caused by repeated evaluations of derivatives and matrix inverses, which can be a constraint in large systems or complex functions. The iterative steps of this method are generally written as (1).

$$x^{(k+1)} = x^k - J^{-1}\left(x^k\right).F\left(x^k\right) \tag{1}$$

In the iteration process, $x^{(k)}$ is the approximation of the solution at iteration k, meaning the approximate value of the solution after the iteration step is performed. The function $F(x^k)$ represents the evaluation of the nonlinear function system at point x^k, indicating how far the current solution is from the zero value of the function. The matrix $J\left(x^k\right)$ is the Jacobian matrix calculated at point x^k, containing the partial derivatives of the system functions with respect to their variables. The inverse of the Jacobian matrix, $J^{-1}\left(x^k\right)$, is used to determine the direction and magnitude of the correction to the solution approximation so that the next iteration is closer to the exact solution. By using this Jacobian inverse, the iterative method can improve the solution efficiently at each step. Equation (1), therefore, denotes the iterative formulation of the Newton-Raphson method, in which the Jacobian is recomputed at each step. This continuous updating enables the method to adjust the correction process dynamically, thereby improving accuracy and accelerating convergence, particularly when the initial approximation is sufficiently near the actual solution.

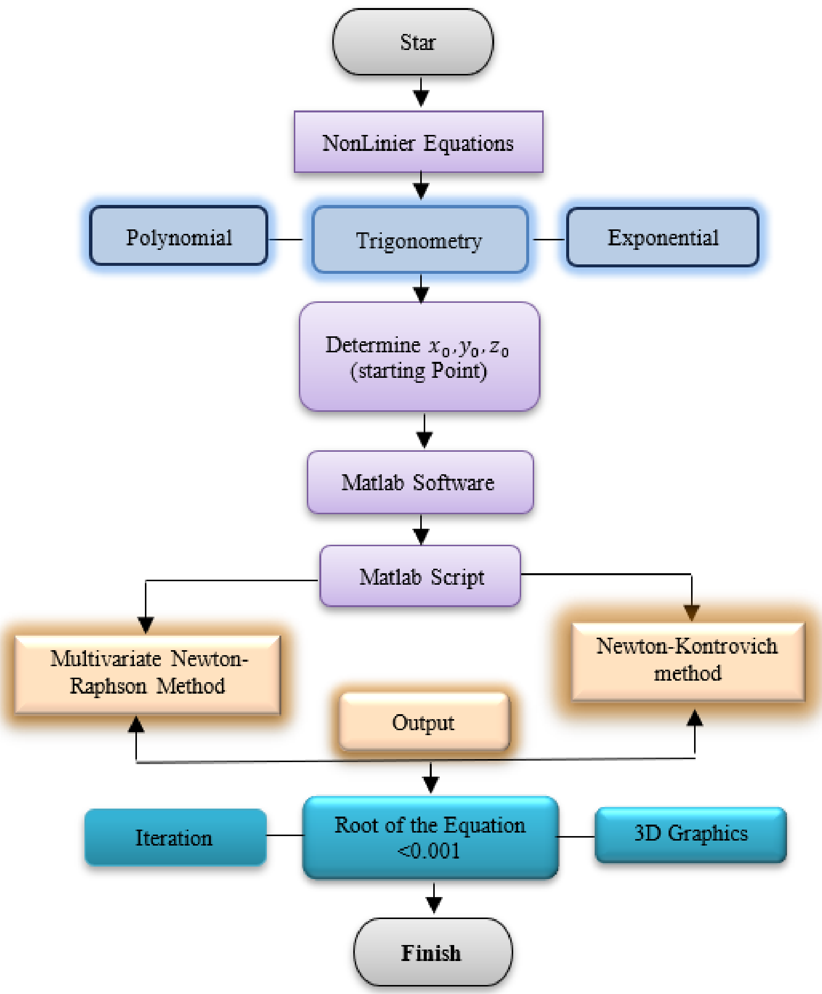

Figure 1. Research Flow

B. Newton-Kontrovich Method

The Newton-Kantorovich method is an iterative method based on a linear approximation of nonlinear systems, similar to Newton-Raphson, but with a theoretically tighter convergence approximation. This method calculates the Jacobian matrix only once at the beginning and uses it consistently throughout the iteration, thereby reducing the computational load, especially in large-scale systems. However, because the Jacobian is not updated, this method is more prone to convergence failure, especially in systems with high nonlinearity or when the initial guess is far from the solution. Thus, the success of the Newton-Kantorovich method depends heavily on the nature and complexity of the problem being solved. This method is also written in (2).

$$x^{(k+1)} = x^k - J^{-1}(x^{(0)}).F(x^k) \tag{2}$$

Where k denotes the iteration step, $x^{(k)}$ is an approximation of the solution obtained at that step. The function $F(x^k)$ is the evaluation of the nonlinear system calculated at point $x^{(k)}$, describing the function value based on the current solution approximation. The Jacobian matrix, which contains the partial derivatives of the system, is calculated at the initial point $x^{(0)}$, and the inverse of this Jacobian matrix, $J^{-1}(x^{(0)})$, is computed only once at the beginning of the iteration process. The use of this fixed Jacobian inverse serves to improve the solution approximation gradually in each iteration until convergence is achieved, thereby reducing the computational load compared with recalculating the Jacobian inverse at each iteration step. Unlike Equation (1), Equation (2) fixes the Jacobian at its initial value. This means the correction direction remains constant throughout the iteration process. While this reduces computational cost by avoiding repeated Jacobian evaluations, it also limits the adaptability of the method to the nonlinear dynamics of the system, often resulting in slower convergence or potential divergence for complex problems.

$$Non\ Linear\ Equation: \begin{cases} f_1\,(x,y,z) = e^y + x^2 - z = 0 \\ f_2\,(\mathrm{x,y,z}) = x^2 + \cos(y) - 3 = 0 \\ f_3\,(\mathrm{x,y,z}) = x^2 + y + z^2 = 0 \end{cases} \quad (3)$$

Both methods are then used to find solutions to nonlinear systems of equations. The system of equations used in the simulation consists of (3). This study followed a series of systematically arranged stages to achieve its objectives. Each step in the process was designed to ensure the accuracy and reliability of the results obtained. The complete procedure for conducting this study can be seen in Figure 1.

Figure 1 shows the systematic stages in the research process aimed at solving nonlinear systems of equations through a MATLAB-based numerical approach. The research begins with the development of algorithms for two iterative methods, the Multivariate Newton-Raphson Method and the Newton-Kantorovich Method, based on the literature review, which are then implemented as MATLAB scripts. The nonlinear system used as the object of study includes polynomial, trigonometric, and exponential components. Graphs of each function were used to determine the initial values of variables x_0, y_0, and z_0 as the starting points for the iterative process. Simulations were conducted by directly inputting the three functions into each method, using an error tolerance of 0.001 and a maximum of 100 iterations. The iterative process continued until the obtained solution met the convergence criteria. The simulation results are observed based on the number of iterations, the root values obtained, the final error, and the stability of the numerical results during the iterative process, and were visualized in the form of a 3D graph as an indicator of the method's convergence success. All these steps reflect an efficient and systematic numerical problem-solving strategy for nonlinear systems of equations with the assistance of MATLAB software. The algorithms were implemented in MATLAB, where each method was coded to perform function evaluation, Jacobian computation, error measurement, and convergence checking.

Table 1 presents the implementation scripts for both methods. The Newton-Raphson script updates the Jacobian matrix at every iteration, while the Newton-Kantorovich script uses the Jacobian fixed at the initial approximation. Both scripts stop when the error falls below the tolerance or when the maximum number of iterations is reached.

TABLE 1
SCRIPT FOR EACH METHOD

<table>
<tr><th>No</th><th>Method</th><th>MATLAB Script</th></tr>
<tr><td>1.</td><td>Newton-Rapshon Multivariat</td><td><pre>for k = 1:max_iter
    F_val = F_func(xk); J_val = J_func(xk);
    delta = -J_val \ F_val;
    xk_next = xk + delta;
    err = norm(delta);
    fprintf('%d\t\t%.6f\t%.6f\t%.6f\t%.6e\n', k, xk_next(1), xk_next(2), xk_next(3), err);
    if err < tol
        break;
    end
    xk = xk_next;
end</pre></td></tr>
<tr><td>2.</td><td>Newton-Kontrovich</td><td><pre>for k = 1:max_iter
    x_val = X(1); y_val = X(2); z_val = X(3);
    F_val = F(x_val,y_val,z_val);
    J_val = J(x_val,y_val,z_val);
    delta = -J_val\F_val;
    X = X + delta;
    err = norm(delta, inf);
    fprintf('%d\t\t%.6f\t%.6f\t%.6f\t%.6e\n', k, X(1), X(2), X(3), err);
    if any(abs(X) > 1e6) || any(isnan(X)) || any(isinf(X))
        fprintf('Solution diverged or invalid. Iteration stopped.\n');
        break;
    end
    if err < tol
        fprintf('Converged in %d iterations.\n', k);
        fprintf('Final solution: x=%.6f, y=%.6f, z=%.6f\n', X(1), X(2), X(3));
        break;
    end
end</pre></td></tr>
</table>

TABLE 2
SIMULATION RESULT

Method	Iteration	X	Y	Z	Error
Newton-Raphson Multivariate	7	-0.907	-1.478	1.307	2.711×10^{-7}
Newton-Kantorovich	21	-1.624	-1.228	1.386	6.770×10^{-5}

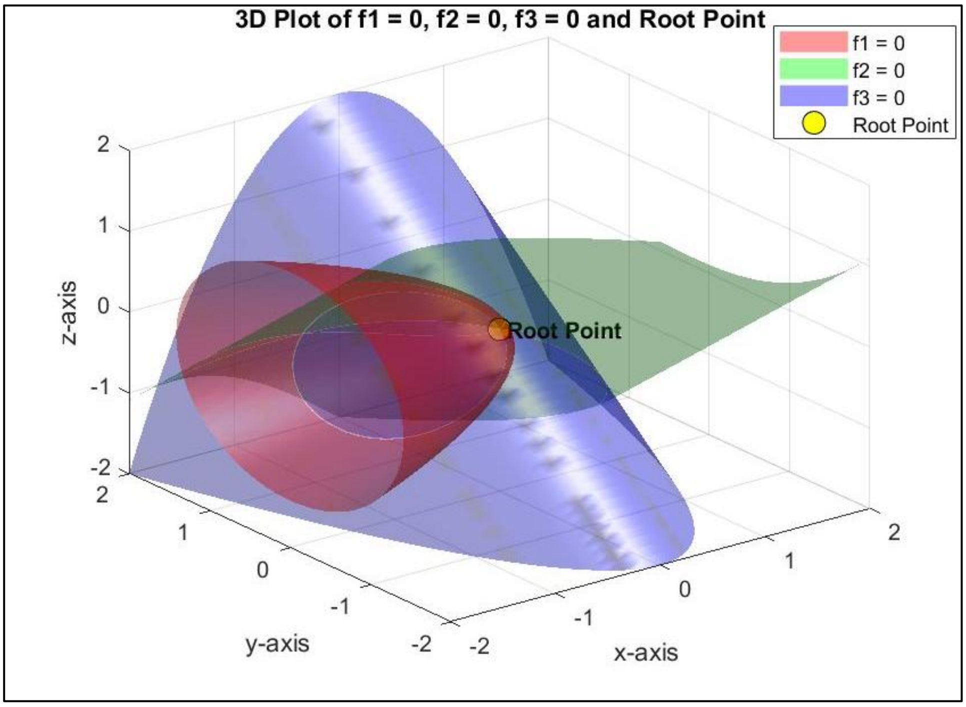

Figure 2. Visualization of the Multivariate Newton-Raphson Method solution

III. RESULTS AND DISCUSSION

A. *Result*

The numerical simulations using both methods produced iteration data that include the number of steps required for convergence, the approximate root values for each variable (x, y, z), and the corresponding error magnitudes. These outcomes serve as the main indicators for assessing the efficiency and accuracy of the Multivariate Newton-Raphson and Newton-Kantorovich methods when applied to the same nonlinear system. To provide a clear overview, the iteration results are first summarized in Table 2, followed by three-dimensional visualizations in Figures 2 and 3 that illustrate the convergence trajectories of each method.

Table 2 shows that the Multivariate Newton-Raphson method provides the best performance in solving the tested nonlinear system of equations. The simulation was performed using the initial point [1, 0, 2], and the computational results show that the Multivariate Newton-Raphson method requires 7 iterations to approach the solution. The simulation results show values of $x =$ -0.907, $y =$ -1.478, and $z =$ 1.307 with an error value of 2.711×10^{-7}. This very small error value indicates that the Multivariate Newton-Raphson method is capable of producing a solution very close to the actual root of the tested system. The low iteration count reflects higher computational efficiency, while the small error value confirms the method's accuracy in approaching the exact solution.

In contrast, the Newton-Kantorovich method required 21 iterations to converge to the same system, with solutions $x =$ -1.624, $y =$-1.228, and $z =$ 1.386. Although it also achieved a relatively small error value of 6.770×10^{-5}, the number of iterations required was more than twice that of the Multivariate Newton-Raphson method. This indicates that the Newton-Kantorovich method tends to adapt more slowly to the nonlinear dynamics of the system, possibly because of its approach of using a fixed or semi-frozen Jacobian in the iteration process. This means that, despite eventually converging, the Newton-Kantorovich method is less accurate and demands substantially more computational effort. The higher iteration count highlights its lower efficiency, while the larger error value shows that its solution is less precise than that of the Multivariate Newton-Raphson method. Overall, Table 2 illustrates that

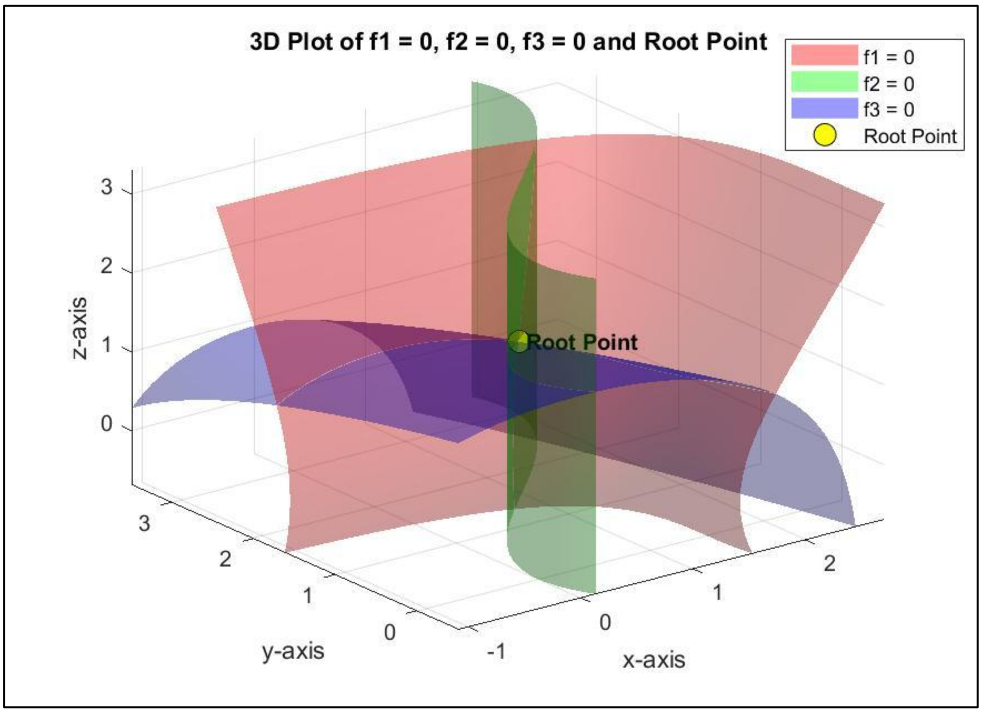

Figure 3. Visualization of the Newton-Kantorovich Method solution

the Multivariate Newton-Raphson method is both more efficient, as measured by the number of iterations, and more accurate, as indicated by the smaller error, compared to the Newton-Kantorovich method. This reinforces the advantage of updating the Jacobian at each step, which enables faster adaptation to the nonlinear dynamics of the system.

Next, a visualization graph containing the solutions to the nonlinear system of equations is presented. The graph below shows the solutions produced by each method to illustrate their characteristics. The three-dimensional visualization shows the iteration path toward the solution, as well as the differences in convergence patterns between the two methods. Figures 2 and 3 present the visualization results of the two methods in sequence.

Figure 2 shows a three-dimensional visualization of the solution to a nonlinear system of equations using the Multivariate Newton-Raphson method. The red surface represents the function $f^1(x, y, z) = 0$, the green surface represents $f_2(x, y, z) = 0$ and the blue surface corresponds to $f_3(x, y, z) = 0$. The intersection point of these three surfaces marks the exact root of the nonlinear system. This simulation shows that the Multivariate Newton-Raphson method converges in 7 iterations toward the root point. The iterative trajectory shown in the figure moves directly and steadily toward the intersection, reflecting the method's ability to adjust its direction adaptively through Jacobian updates at each step. As a result, convergence is faster and the solution is highly accurate in three-dimensional space.

Figure 3 shows a three-dimensional visualization of the simulation results of a nonlinear system of equations using the Newton-Kantorovich method. Similar to Figure 2, the red, green, and blue surfaces represent the functions $f_1(x, y, z) = 0, f_2(x, y, z) = 0$, and $f_3(x, y, z) = 0$, respectively, and their intersection denotes the root of the system. In this simulation, the Newton-Kantorovich method required up to 21 iterations to converge to the root point. Unlike the Newton-Raphson trajectory, the iteration path here is less direct, involving more detours and adjustment steps before approaching the intersection. This reflects the limitation of using a fixed Jacobian, which slows the method's ability to adapt to changes in the function contour. Consequently, the convergence rate is slower and less efficient, even though the method eventually identifies a solution near the root.

These results indicate that the Multivariate Newton-Raphson method demonstrates higher efficiency than the Newton-Kantorovich method, particularly in terms of the number of iterations required and the smaller error values achieved. While Newton-Kantorovich shows greater stability in certain cases, its slower convergence makes it less practical for the tested nonlinear system. To validate and contextualize these findings, the subsequent discussion provides a comparative analysis with previous studies that have applied similar methods, highlighting both consistencies and differences with earlier results.

B. Discussion

The results of the study show that the Multivariate Newton-Raphson method performs better than the Newton-Kantorovich method in terms of both convergence speed and accuracy. With only 7 iterations and a very small error value, the Newton-Raphson method produces a highly accurate solution. In contrast, the Newton-Kantorovich method requires more than twice as many iterations, namely 21, and produces a larger error. This slower convergence is caused by the use of a fixed or semi-frozen Jacobian, which limits the method's ability to adapt to the nonlinear dynamics of the system.

Three-dimensional visualization further confirms these findings. The Newton-Raphson method shows a more direct and efficient convergence path to the root point, while the Newton-Kantorovich method requires more correction steps, making it less efficient even though it eventually converges. This shows that Multivariate Newton-Raphson is more adaptive in following changes in the function contour, while Newton-Kantorovich tends to be slow to respond, requiring longer computation time.

Research conducted by [29] on the efficiency of iterative methods shows that Multivariate Newton-Raphson requires only 12 iterations to achieve a certain error tolerance, while Newton-Kantorovich requires 24 iterations, which is 50% more. In addition, [30], in their study on the stability of the Newton-Kantorovich method, concluded that although this method has strong semi local convergence guarantees, its speed is about 60–80% slower than Newton-Raphson. Research by [31] also reinforces that although the gamma-condition can improve the stability of Newton-Kantorovich, this method still requires 30–40% more iterations than Newton-Raphson. These findings highlight the efficiency advantage of the Multivariate Newton-Raphson method, while the Newton-Kantorovich method offers stronger initial convergence stability.

The finding that the Newton-Raphson method converges faster is consistent with the results reported by [32] and [33], which also highlighted its efficiency in low-dimensional nonlinear systems. However, unlike the present study, those works did not directly compare Newton-Raphson with Newton-Kantorovich under identical conditions. Our results extend these insights by showing that although Newton-Kantorovich offers theoretical stability advantages, as discussed by [34], in practice its performance is less efficient when applied to the tested nonlinear system. This direct comparison contributes to the literature by explicitly demonstrating the trade-off between efficiency and stability across the two methods within the same experimental framework.

Overall, this study strengthens the evidence that Newton-Raphson remains the most efficient method for nonlinear systems when convergence speed and accuracy are prioritized. At the same time, it clarifies the limitations of Newton-Kantorovich in practical computation, thereby providing a more balanced perspective for researchers when selecting iterative methods for solving nonlinear equations.

IV. CONCLUSION

Based on the simulation and numerical analysis results, the Multivariate Newton-Raphson method proved to be the more efficient and accurate approach for solving three-variable nonlinear systems of equations. This method successfully achieved a solution with a very small final error rate of 1.219×10^{-9} in only 7 iterations. The convergence obtained is also stable because the Jacobian is updated at each step, allowing the iteration direction to adjust optimally to the function contour. In contrast, the Newton-Kantorovich method requires up to 21 iterations with a larger final error of 9.770×10^{-7} because it uses a fixed Jacobian, which leads to a slower convergence rate. Visualization of the convergence trajectory supports these findings, as the Newton-Raphson method demonstrates a more direct and efficient path to the solution. Therefore, this method is recommended as the primary choice for solving nonlinear systems that require high accuracy and efficiency. Further research is recommended to test the performance of both methods on nonlinear systems with different characteristics, such as non-smooth functions, large systems, or systems with initial values far from the root. Exploring hybrid approaches that combine the convergence speed of Newton-Raphson with the stability of Newton-Kantorovich also has the potential to produce more adaptive solutions. Additionally, developing interactive 3D graphical visualizations can provide a more intuitive understanding of convergence dynamics and help identify the behavior of numerical methods in response to variations in system structure and initial parameters.

DECLARATION OF AI AND AI ASSISTED TECHNOLOGIES IN THE WRITING PROCESS

During the preparation of this work the authors used AI services in order to write the manuscript.

After using these services, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

CREDIT AUTHORSHIP CONTRIBUTION STATEMENT

Syaharuddin: Conceptualization, Investigation, Project administration, Resources, and Software. **Hendi Hidayah**: Conceptualization, Supervision, Validation, and Writing – review & editing. **Mahsup**: Supervision, Validation, and Writing – review & editing. **Saba Mehmood**: Supervision, Validation, and Writing – review & editing. **Wasim Raza**: Supervision, Validation, and Writing – review & editing.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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REFERENCES

[1] M. Shakeel, S. T. Mohyud-Din, and M. A. Iqbal, "Modified extended exp-function method for a system of nonlinear partial differential equations defined by seismic sea waves," *Pramana - J. Phys.*, vol. 91, no. 2, pp. 1–8, 2018.

[2] V. Torkashvand, "A two-step method adaptive with memory with eighth-order for solving nonlinear equations and its dynamic," *Comput. Methods Differ. Equations*, vol. 10, no. 4, pp. 1007–1026, 2022.

[3] J. Ríos-Ocampo and M. S. Gary, "Using analytical equations to represent nonlinear relationships," *Syst. Dyn. Rev.*, vol. 38, no. 4, pp. 354–370, 2022.

[4] L. Zakaria, A. Eka, and D. Aziz, "Penyelesaian Sistem Persamaan Fully Fuzzy Non Linear Menggunakan Metode Newton Raphson Ganda," *J. Math. Theory Appl.*, vol. 5, no. 2, pp. 67–73, 2023.

[5] P. Batarius, J. San Juan, and P. Kupang, "Perbandingan Metode Newton-Raphson Modifikasi Dan Metode Secant Modifikasi Dalam Penentuan Akar Persamaan," *Semin. Nas. Ris. dan Teknol. Terap.*, vol. 8, no. Ritektra 8, pp. 53–63, 2018.

[6] E. Sunandar and I. Indrianto, "Perbandingan Metode Newton-Raphson & Metode Secant Untuk Mencari Akar Persamaan Dalam Sistem Persamaan Non-Linier," *J. Pengkaj. dan Penerapan Tek. Inform.*, vol. 13, no. 1, pp. 72–79, 2020.

[7] F. Ahmad, "Multi-step derivative-free preconditioned Newton method for solving systems of nonlinear equations," *SeMA J.*, vol. 75, no. 1, pp. 45–56, 2018.

[8] M. Adriana, "on the Convergence of the Newton-Raphson Method and Some of Its Generalizations," *Bull. Transilv. Univ. Brasov, Ser. III Math. Comput. Sci.*, vol. 4, no. 2, pp. 215–224, 2024.

[9] M. Putri and S. Syaharuddin, "Implementations of Open and Closed Method Numerically: A Non-linear Equations Solution Convergence Test," *IJECA (International J. Educ. Curric. Appl.*, vol. 2, no. 2, p. 1, 2019.

[10] Inderjeet and R. Bhardwaj, "A new Iterative Newton Raphson technique for the numerical simulation of Nonlinear Equations," *J. Integr. Sci. Technol.*, vol. 13, no. 4, pp. 1–6, 2025.

[11] M. Mohammad Ali, "Numerical differential continuation approach for systems of nonlinear equations with singular Jacobian," *AUT J. Math. Comput.*, vol. 3, no. 1, pp. 53–58, 2022.

[12] Y. Liao and L. Cui, "Newton Like Iterative Method without Derivative for Solving Nonlinear Equations Based on Dynamical Systems," *J. Res. Sci. Eng.*, vol. 5, no. 5, pp. 1–7, 2023.

[13] A. Hasanudin, "Calculation of Loan Amount When Cooperatives Do Not Make Profits with Non-Linear Equation Method Using Secant and Its Implementation with MATLAB," *J. Penelit. Mat. dan Pendidik. Mat.*, vol. 8, no. 1, pp. 17–22, 2023.

[14] S. Regmi, I. K. Argyros, S. George, and M. I. Argyros, "A Comparison Study of the Classical and Modern Results of Semi-Local Convergence of Newton–Kantorovich Iterations-II," *Mathematics*, vol. 10, no. 11, pp. 1–14, 2022.

[15] Yudhi, Devitriani, Mariatul Kiftiah, "Analisis Metode Newton-Raphson Ganda Orde Konvergensi Empat Dalam Menyelesaikan Sistem Persamaan Nonlinear," *Bimaster Bul. Ilm. Mat. Stat. dan Ter.*, vol. 8, no. 2, pp. 213–220, 2019.

[16] J. R. Sharma and H. Arora, "Improved Newton-like methods for solving systems of nonlinear equations," *SeMA J.*, vol. 74, no. 2, pp. 147–163, 2017.

[17] O. Paulina Maure and H. Tulan, "Studi Komparasi Beberapa Metode Numerik Dalam Mengaproksimasi Akar-Akar Persamaan Non Linear," *Asimtot J. Kependidikan Mat.*, vol. 6, no. 01, pp. 1–12, 2024.

[18] J. Lacotte and M. Pilanci, "Adaptive and Oblivious Randomized Subspace Methods for High-Dimensional Optimization: Sharp Analysis and Lower Bounds," *IEEE Trans. Inf. Theory*, vol. 68, no. 5, pp. 3281–3303, 2022.

[19] M. Farman, A. Akgül, N. Alshaikh, M. Azeem, and J. Asad, "Fractional-Order Newton–Raphson Method for Nonlinear Equation With Convergence and Stability Analyses," *Fractals*, vol. 31, no. 10, pp. 1–12, 2023.

[20] L. F. Grisales-Noreña, O. D. Montoya, W. J. Gil-González, A. J. Perea-Moreno, and M. A. Perea-Moreno, "A comparative study on power flow methods for direct-current networks considering processing time and numerical convergence errors," *Electron.*, vol. 9, no. 12, pp. 1–20, 2020.

[21] H. Okawa *et al.*, “The W4 method: A new multi-dimensional root-finding scheme for nonlinear systems of equations,” *Appl. Numer. Math.*, vol. 183, no. 1, pp. 157–172, 2023.

[22] S. Regmi, I. K. Argyros, S. George, and M. I. Argyros, “Developments on the Convergence Analysis of Newton-Kantorovich Method for Solving Nonlinear Equations,” *Eur. J. Math. Anal.*, vol. 3, no. 11, p. 15, 2023.

[23] S. Regmi, I. K. Argyros, S. George, and J. Warden, “A Unified Kantorovich-type Convergence Analysis of Newton-like Methods for Solving Generalized Equations under the Aubin Property,” *Eur. J. Math. Anal.*, vol. 4, no. 9, p. 3, 2024.

[24] A. G. Kamel, E. H. Haraz, and S. N. Hanna, “Numerical Simulation of Channel Flow Over a Skewed Equilateral Cavity,” *J. Appl. Math. Comput. Mech.*, vol. 19, no. 3, pp. 29–43, 2020.

[25] N. K. Vitanov, “Simple Equations Method (SEsM): An Effective Algorithm for Obtaining Exact Solutions of Nonlinear Differential Equations,” *Entropy*, vol. 24, no. 11, pp. 1–55, 2022.

[26] I. K. Argyros, S. George, S. Shakhno, S. Regmi, M. Havdiak, and M. I. Argyros, “Asymptotically Newton-Type Methods without Inverses for Solving Equations,” *Mathematics*, vol. 12, no. 7, pp. 1–19, 2024.

[27] R. A. Isewid, “An Enhanced Newton-Kantorovich Technique for Nonlinear Problem-solvers to Optimal Control Convergence,” *Al-Qadisiyah J. Pure Sci.*, vol. 29, no. 20, pp. 131–140, 2024.

[28] E. Weinan, J. Han, and A. Jentzen, “Deep Learning-Based Numerical Methods for High-Dimensional Parabolic Partial Differential Equations and Backward Stochastic Differential Equations,” *Commun. Math. Stat.*, vol. 5, no. 4, pp. 349–380, 2017.

[29] H. Ali, T. Datta, and M. Kamrujjaman, “Efficient Family of Iterative Methods for Solving Nonlinear Simultaneous Equations: A Comparative Study,” *J. Appl. Math. Comput.*, vol. 5, no. 4, pp. 331–337, 2021.

[30] H. A. H. Abugirda, K. S. Al-Yasiri, and M. K. Abdullah, “Newton-Kantorovich Method for Solving One of the Non-Linear Sturm-Liouville Problems,” *Baghdad Sci. J.*, vol. 20, no. 3, pp. 2036–2041, 2023.

[31] A. Ivanov, “On transversal connecting orbits of Lagrangian systems in non-stationary force field: Newton-Kantorovich approach,” *IEEE Technol. a Sustain. Clim. Collect.*, vol. 26, no. 392, pp. 1–27, 2019.

[32] Ľ. Baňas and S. Herr, “Numerical approximation of bi-harmonic wave maps into spheres,” *SIAM J. Numer. Anal.*, vol. 63, no. 3, pp. 1–20, 2025.

[33] E. Berglund, S. Khirirat, and X. Wang, “Zeroth-Order Randomized Subspace Newton Methods,” *u IEEE Int. Conf. Acoust. Speech Signal Process.*, vol. 9, no. 4, pp. 1–9, 2022.

[34] I. K. Argyros, S. Regmi, S. Shakhno, and H. Yarmola, “A Methodology for Obtaining the Different Convergence Orders of Numerical Method under Weaker Conditions,” *Mathematics*, vol. 10, no. 16, pp. 19–21, 2022.